Microwave imaging using a Time Reversal RADAR


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Frame of the study

**Inverse scattering for imaging in random media**

Aim at taking benefit from the ability of **selectively focusing** onto a target to improve the **robustness** of inversion algorithms against **clutter**
Summary

- Building a focusing wave: Time Reversal and D.O.R.T. methods
- Experimental focusing wave generation
- Experimental inversion
- Taking advantage of D.O.R.T. fields
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The goal is to focus a maximum amount of energy onto a target. A sequence of illuminations is used, such that each incident wave is the time-reversed replica of the previous measured scattered wave.
Properties

Wave propagation in a homogeneous medium is ruled by d'Alembert equation:

\[
\left( \Delta - \frac{1}{c_b^2} \frac{\partial^2}{\partial t^2} \right) w(\vec{r}, t) = 0
\]

Both \( w(\vec{r}, t) \) and \( w(\vec{r}, -t) \) are solutions

In the frequency domain, both \( W(\vec{r}, f) \) and \( W^*(\vec{r}, f) \) are solutions → Phase Conjugation

Resolution limit

\[
\frac{\lambda_b}{2} \quad \frac{\lambda_b F}{D}
\]

\[
\frac{D}{F}
\]
D.O.R.T. method

What if more targets?

$(K^H K)^n e(0) = (V \Lambda V^H)^n e(0) \approx \lambda_1^n < v_1, e(0) > v_1$

allow to **selectively focus** onto each target and **without iterations**!

- Independent point-like scatterers (scalar theory)
- Scatterer dimension comparable to wavelength: couplings, symmetric/antisymmetric eigenvalues, ...

$\Rightarrow (\lambda_i, v_i)$

$\rightarrow K^H K e(0)$

$\leftarrow K^T K e(0)$

$\rightarrow K e(0)$

$\leftarrow e(0)$
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UWB Phase Conjugation prototype

A = attenuators
- Dynamic Range > 30 dB
- Resolution ~ 0.11 dB/code

Φ = phase shifters
- Dynamic Range ~ 360 deg
- Resolution ~ 0.36 deg/code

E.T.S.A. antennas
- SWR < 2 in [2-18] GHz

RF section
- Antennas (A)
- Switches
- Phase Shifters (Φ)
- Power Splitter
- PIC + Interface
- Vector Network Analyzer
- ETHERNET
- USB

[2-4] GHz
- TM pol (E//)

Spare Antenna (TX/RX)
1) DATA ACQUISITION

Time domain
Field chart

Frequency domain
Backpropagated phase

Time domain
Max. amplitude vs. A9 position

2) BACK-PROPAGATION

A9 channels

Multiplexer

Antenna array

Normalized Electric Field

Cross-range (m)

Amplitude norm. (dB)

A9 position (cm)
Focusing wave: DORT experiment

Frequency domain
Singular values vs. frequency

Time domain
Field chart → $(\lambda_1, v_1)$

Frequency domain
Field chart → $(\lambda_2, v_2)$ @ 3 GHz
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Inversion – Incident field calibration

- Need to know the incident field in the investigation domain
- Not really far-field conditions, antennas mismatched, coupled antennas
  - Need for measured data in the actual configuration

Multipolar expansion of the incident field
+ Split into TX/RX antenna effective lengths

Measured scattered field (Amplitude, Phase)

Synthetic scattered field (Amplitude, Phase)
2D Inversion – Modified gradient algorithm

- Domain integral formalism ($\Omega$)
- Minimization of a cost function $\mathcal{F}_n$
- Multisource ($j$), multifrequency ($p$)
- Contrast positivity constraint
- Initial guess: backpropagation

Unknows
- Contrast: $\chi_p = |\xi|^2 - i \frac{\eta^2}{\omega_p \varepsilon_0}$
- Total field in the domain: $E_{j,p}$

Observations: $E_{\text{d};\text{meas}}$

\[ E_{\text{d};\text{meas}} = G \Gamma_{j,p} \chi_p E_{j,p} \] (state eq.), \[ E_{j,p} = E^i + G \Omega_{j,p} \chi_p E_{j,p} \] (field eq.)

\[ \mathcal{F}_n(E_{j,p}; \chi_p) = W_\Gamma \sum_{j=1}^J \sum_{p=1}^P \|h^{(1)}_{j,p}\|^2_\Gamma + W_\Omega \sum_{j=1}^J \sum_{p=1}^P \|h^{(2)}_{j,p}\|^2_\Omega \]

\[ h^{(1)} = E_{\text{d};\text{meas}} - G \Gamma_{j,p} \chi_p E_{j,p}, \quad h^{(2)} = E_{j,p} - E_{j,p} + G \Omega_{j,p} \chi_p E_{j,p} \]

\[ \begin{align*}
\xi_n &= \xi_{n-1} + \alpha^\xi_n d^\xi_n, \\
\eta_n &= \eta_{n-1} + \alpha^n_n d^n_n, \\
E_n &= E_{n-1} + \alpha^E_n d^E_n + \gamma_n w_n
\end{align*} \]

$d^\xi_n, d^n_n, d^E_n$: conjugate gradient descent directions

$w_n = E_{n-1} - \tilde{E}_{n-1}, \quad \tilde{E}_{n-1} = (1 - G \Omega \chi_{n-1})^{-1} E^i$ (Born method)
Inversion – Results

Buried object configuration
- Reflection data
- Metal target
- Not really a 2D target...

(Very) limited aspect configuration!

Experimental

Synthetic data

Initial estimate

Final result

15/24
Inversion – Results

Transm+Reflec configuration - Metal + Wood targets

Initial estimate

Final result

\(\text{Re}(\epsilon_r)\)

\(\sigma\)
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Algorithm including D.O.R.T. fields

Idea 1: Instead of using the “regular” incident field $E^i$, use the D.O.R.T. incident field $E^{i;DORT}$, which focuses onto the target and gives a scattered field with a better signal to clutter ratio

$$\mathcal{F} \rightarrow \mathcal{F}^{DORT}$$

Idea 2: Use $\mathcal{F}^{DORT}$ as a regularization term

$$\mathcal{F} \rightarrow \mathcal{F} + \kappa^2 \mathcal{F}^{DORT}$$
$$\mathcal{F} \rightarrow \mathcal{F} \times \mathcal{F}^{DORT}$$
Circular scanner configuration (synthetic data)

- Monochromatic data
- 2D
- TM pol ($E_{//}$)

D.O.R.T. focusing waves from quarter-circle sectors
Circular scanner configuration (synthetic data)

Inversion

- no clutter (64 waves)
- without DORT: $\mathcal{F}$ (64 waves)
- with DORT: $\mathcal{F}^\text{DORT}$ (4 focusing waves)
Buried object (synthetic data)

- ✔ Monochromatic data
- ✔ Frequency hopping inversion
- ✔ 2D
- ✔ TM pol ($E_{\parallel}$)

Diagram:

Source

$\varepsilon_0, \mu_0$

$\Gamma$

Receivers

$h_1 = 0.75\text{m}$

$\varepsilon = 4$

$\varepsilon_r(x,y) = \varepsilon_2(1 + 0.15r(x,y))$

$\varepsilon_r = 7$

$a_1 = 0.15\text{m}$

$h_2 = 0.95\text{m}$

$\varepsilon_r = 6$

$a_2 = 0.1\text{m}$

$\Omega$
Buried object (synthetic data)

D.O.R.T. focusing waves

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Signal/Clutter</th>
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<tbody>
<tr>
<td>100</td>
<td>9.2 dB</td>
</tr>
<tr>
<td>200</td>
<td>5.5 dB</td>
</tr>
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Buried object (synthetic data)

Inversion

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without DORT:

with DORT: $F \times \mathcal{F}_{DORT}$
Conclusions and future work

- The prototype works well in generating a focusing wave
- Array mall aperture angle is the bottleneck
- D.O.R.T. exploitation is promising (synthetic data) for increasing robustness of the inversion
  - if “rich” D.O.R.T.incidences available, using $\mathcal{F}^{\text{DORT}}$ is sufficient
  - otherwise, multiplicative regularization seems OK

- Acquisition of a second array to measure the response to the D.O.R.T. focusing wave
- Implementing D.O.R.T. multiplicative regularization in the multifrequency frame
Inversion – Incident field calibration

1) Measure $S_{jg}^{\text{cal}}$ and retrieve the effective length $l_e(\theta_{jg})$ for each antenna ($A_1..A_8$)

$$S_{jg}^{\text{cal}} = \omega \mu_0 \frac{e^{-ikr_{jg}}}{r_{jg}} l_e^2(\theta_{jg}) \Rightarrow l_e(\theta_{jg}), j = 1, N_{\text{cal}}$$

2) TX: express the incident field as:

$$E_k^i = \omega \mu_0 \sum_{k=-N_{\text{mul}}}^{N_{\text{mul}}} \gamma^k_{n,tx} H_n^-(kr_k)e^{-in\theta_k}, \quad k = 1, N_{\text{tx}}$$

whose coef’s are found by solving:

$$l_e(\theta_{jg})\frac{e^{-ikr_{jg}}}{r_{jg}} = \sum_{k=-N_{\text{mul}}}^{N_{\text{mul}}} \gamma^k_{n,tx} H_n^-(kr_k)e^{-in\theta_{jg}}$$

3) RX: express the receiving Green function as:

$$G_m^\Gamma = \sum_{k=-N_{\text{mul}}}^{N_{\text{mul}}} \gamma^k_{n,rx} H_n^-(kr_m)e^{-in\theta_m}, \quad k = 1, N_{\text{rx}}$$

whose coef’s are found by solving:

$$l_e(\theta_{jg})H_0^-(kr_{jg}) = \sum_{k=-N_{\text{mul}}}^{N_{\text{mul}}} \gamma^k_{n,rx} H_n^-(kr_{jg})e^{-in\theta_{jg}}$$
Buried object (synthetic data)

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- Without DORT nor positivity
- Without DORT but positivity
- With DORT and positivity
Non-uniqueness of the solution

\[\chi = \varepsilon - \varepsilon_2\]

If \(\tilde{\chi} = -\chi\) and \(\tilde{d} = d \pm \frac{\lambda_2}{2}\)

\[\Rightarrow\text{ we have the same scattered field: } \tilde{E}^d = E^d\]